Dynamic Auctions with Bank Accounts
Implementing Bundling in an Online Fashion

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The seller has a sequence of items to sell to a single buyer.

The items arrive over time.
  - At each stage, there is one item for sale.
  - The item will be destroyed at the end of this stage, if not sold.

Nobody knows the actual value of the $t$-th item until the beginning of the $t$-th stage.
  - Independent valuations, commonly known priors.

The seller’s allocation rule and payment rule could depend on past stages.
An Application: Ads

- Google/Baidu/Bing sells ad impressions to advertisers.
- Impressions may come from users’ searches on search engines. (Arrive over time, destroyed immediately if not sold.)
- The value of each impression varies with (at least) the user’s information (location, time, age, gender, cookies, etc.).
- Currently, the auctions are rarely conducted dynamically.
<table>
<thead>
<tr>
<th>auction-based</th>
<th>dynamic</th>
<th>contract-based</th>
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</thead>
<tbody>
<tr>
<td>real-time</td>
<td>real-time + higher revenue</td>
<td>bundling higher revenue</td>
</tr>
<tr>
<td>lack of competition</td>
<td>complicated + commitment power</td>
<td>high entering cost</td>
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<tr>
<td>lower revenue</td>
<td></td>
<td>not real-time</td>
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We introduce a family of *simple* dynamic auctions — coined *bank account mechanisms* to get around these two issues.
Bank Account Mechanism

Stage Mechanism $M_t = \langle z_t, q_t \rangle$

- Buyer: observes $v_t$ and reports it to $M_t$
- Seller: selects $\langle z_t, q_t \rangle$ based on the balance

Bank Account Balance

- Increases
- Decreases

- Seller: spends $s_t$ from the bank account
- Buyer: deposits $d_t$ into the bank account
A Toy Example

Example

- One buyer, two stages, i.i.d. valuations: \( v_1, v_2 \sim F \).
- \( F: \Pr[v = 1] = \Pr[v = 2] = 1/2. \)

Dynamic Auction

- Stage 1: sell the first item at price 2.5.
- Stage 2: allocate the second item if and only if the first item was sold.

Bank Account Mechanism

- Seller sets \( M_1 = \) posted-price at 1;
- Buyer chooses to buy or not,
  and deposits 1.5, if brought;
- if balance = 1.5, Seller spends 1.5,
  and sets \( M_2 = \) give-for-free;
  otherwise, \( M_2 = \) not-for-sale.
Dynamic Bundle

Revenue Comparison

<table>
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<th>auction-based</th>
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<tbody>
<tr>
<td>2</td>
<td>2.5</td>
<td>3</td>
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Bank Account Mechanism interpretation:

- upon Buyer reporting, Seller sells a “dynamic bundle”,
  - **item + item(s)** static bundle
  - **item + future benefits** dynamic bundle
- “future benefits” sold via spends, implemented as discounts in the next stage mechanism.
- Selling “future benefits” brings *the uncertainty of deficits*. 

Mirrokni, Paes Leme, Tang, Zuo
Constraints on Dynamic Bundles.

- The “future benefits” must satisfy certain properties to ensure the mechanism being *incentive compatible* (IC) and *individually rational* (IR).
- Dominant Strategy IC: no difference with the static environment.
- This paper: Bayesian IC + interim IR.
- [Mirrokni, et al. 2016]: Bayesian IC + ex-post IR.
- [Papadimitriou, et al. 2016]: first paper on this setting, discrete and correlated types, focus on complexity.
Bank account mechanism is extremely simple.

**Theorem (Optimal revenue is achievable)**

For any dynamic auction (full history) $M$, there is a (constructive) bank account mechanism that is as good as $M$ for the buyer and is (weakly) better than $M$ for the seller.

Bank account structure identifies trade-offs between revenue and deficits:

**Theorem (Extra revenue comes from dynamic bundles)**

The optimal revenue of a bank account mechanism is bounded by the optimal revenue of static/history-independent mechanisms plus its expected spends, $E \left[ \sum_t s_t \right]$.

Maximum limits on balance imply trade-offs between revenue and deficits.
Due to time limit, we skip this part. See you in the poster session.

- More practical subset of auctions:
  - Deterministic allocations.
  - No payment if nothing gets allocated.
- Extremely simple and easy to describe.
- Efficiently computable OPT vs nearly optimal heuristic.
- Empirical evaluation.

Thanks for your attend!

Thanks! & Questions?
Extra Section

1 Dynamic Auctions

2 Bank Account Mechanisms

3 Double Reserve Auctions
More practical subset of auctions

Some properties are critical in application.

- Deterministic allocations.
- No payment if nothing gets allocated.

Double Reserve Auction (DRA)

- At each stage, it runs a posted-price auction.
- If the item at the previous stage was sold, *low* posted-price for current stage; otherwise, *high* posted-price.

The DRA is extremely simple and easy to describe.
Theorem (Computation of Optimal Double-Reserve Auction)

The optimal double-reserve auction could be computed via a dynamic program. FPTAS for multiplicative revenue approximation.

In contrast of the optimal ones, we propose heuristic DRAs, which are

- easy to construct — only need i) the Myerson reserve and ii) at most two queries to the integration oracle for each stage;
- nearly optimal for various distributions.
Empirical Analysis

HDR vs. OPT on Exponential Distribution

HDR vs. OPT on Lognormal Distribution

HDR vs. OPT on Uniform Distribution
Dynamic mechanisms have great potential to improve the revenue of auctions being repeated over time.

However, the framework in practice is simply history independent or contract-based.

Two major issues for dynamic mechanisms:
- too complicated,
- need strong future commitment power.

Methodology contribution: “bank account mechanism” solves both of these two issues:
- simple structure — only need a “bank account”;
- trade-offs between revenue and commitment power — maximum limit on the balance.

More practical mechanism — (heuristic) double-reserve auction.